

④ a) i) $\sin(2x) = 2\sin(x)\cos(x)$
 ii) $\cos(2x) = 2\cos^2(x) - 1$

b) $\sin(2x) - \tan(x) = \tan(x)\cos(2x)$ Make RHS = LHS

$2\sin(x)\cos(x) - \frac{\sin(x)}{\cos(x)} =$ ~~$\frac{\sin(x)(2\cos^2(x) - 1)}{\cos(x)}$~~

$= \frac{2\sin(x)\cos^2(x)}{\cos(x)} - \frac{\sin(x)}{\cos(x)} = \frac{2\sin(x)\cos^2(x) - \sin(x)}{\cos(x)}$

$= \frac{\sin(x)}{\cos(x)} (2\cos^2(x) - 1) = \tan(x)\cos(2x)$

c) $\sin(2x) - \tan(x) = 0 \rightarrow \tan(x)\cos(2x) = 0$

$\tan(x) = 0$
 $x = 180^\circ$

$\cos(2x) = 0$

$\cos t = 0$
 $t = 90, 270$

$0 < x < 360$
 $t = 2x$

$0 < t < 720$

$x = 45, 135, 225, 315, 180$

$t = 90, 270, 450, 630$
 $x = 45, 135, 225, 315$

⑤ a) $y^2 - xy + 3x^2 - 5 = 0$

$x=1$ $\rightarrow y^2 - y + 3 - 5 = 0$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \rightarrow y = 2 \text{ and } y = -1$

b) i) $y^2 - \boxed{xy} + 3x^2 - 5 = 0$

$2y \frac{dy}{dx} - y - x \frac{dy}{dx} + 6x = 0$

$2y \frac{dy}{dx} - x \frac{dy}{dx} = y - 6x$

$\frac{dy}{dx} (2y - x) = y - 6x$

$\frac{dy}{dx} = \frac{y - 6x}{2y - x}$

xy product rule:

$u = x \quad v = y$

$\frac{d}{dx} = 1 \quad \frac{dy}{dx} = \frac{dy}{dx}$

$\frac{dy}{dx} = y + x \frac{dy}{dx}$

ii) $x=1, y=2 \rightarrow \frac{dy}{dx} = \frac{2-6}{4-1} = -4/3$

$x=1, y=-1 \rightarrow \frac{dy}{dx} = \frac{-1-6}{-2-1} = 7/3$

iii) $\frac{dy}{dx} = 0 \rightarrow \frac{y-6x}{2y-x} = 0 \rightarrow y-6x = 0 \rightarrow y = 6x$

SUB INTO EQUATION: $(6x)^2 - x(6x) + 3x^2 - 5 = 0$

$36x^2 - 6x^2 + 3x^2 - 5 = 0 \rightarrow 33x^2 - 5 = 0$

(6) a) i) $\vec{OB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \therefore \vec{OC} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$

ii) $\vec{AB} = \vec{AO} + \vec{OB} = \begin{pmatrix} -2 \\ -4 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

b) i) $AC = \sqrt{(6-2)^2 + (4-4)^2 + (-2-1)^2}$
 $= \sqrt{25} = 5$

ii) $\vec{AB} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ $\vec{AC} = \vec{AO} + \vec{OC} = \begin{pmatrix} -2 \\ -4 \\ -1 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$

$\vec{AB} \cdot \vec{AC} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} = 4 + 0 + 6 = 10$

$|\vec{AB}| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$

$|\vec{AC}| = 5$

$|\vec{a}| |\vec{b}| \cos(\theta) = \vec{a} \cdot \vec{b}$

$3 \times 5 \times \cos(\theta) = 10$

$\cos(\theta) = 10/15 \rightarrow \theta = \cos^{-1}(2/3) = 48.189\dots = 48^\circ$

c) $\vec{BP} = \vec{BO} + \vec{OP} = \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \alpha-3 \\ \beta-2 \\ \gamma+1 \end{pmatrix}$

For perpendicular, $\vec{a} \cdot \vec{b} = 0 \rightarrow \begin{pmatrix} \alpha-3 \\ \beta-2 \\ \gamma+1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix} = 0$

$\rightarrow 4(\alpha-3) + 0 - 3(\gamma+1) = 0$

$4\alpha - 12 - 3\gamma - 3 = 0$

$4\alpha - 3\gamma = 15$

(7) $\frac{dy}{dx} = 6xy^2 \rightarrow \int \frac{1}{y^2} dy = \int 6x dx$

$\rightarrow -1/y = 3x^2 + C$

$y=1, x=2 \rightarrow -1 = 3(2)^2 + C \rightarrow -1 = 12 + C \rightarrow C = -13$

$\rightarrow -1/y = 3x^2 - 13$

$-1 = y(3x^2 - 13)$

$\frac{-1}{3x^2 - 13} = y$

$y = \frac{1}{13 - 3x^2}$

9) a) i) $\frac{dx}{dt} \propto x(5000-x)$ The number not infected

$$\frac{dx}{dt} = kx(5000-x)$$

ii) $t=0$, $\frac{dx}{dt} = 200$, 1000 rabbits infected

$$\rightarrow 200 = k \times 1000 \times 4000$$

$$200 = 4,000,000k \rightarrow k = 0.00005$$

b) i) $t = 4 \ln \left(\frac{4 \times 2500}{5000 - 2500} \right)$

$$= 4 \ln(4) = 5.5451 \dots = 5.5 \text{ hours (1dp)}$$

ii) $30 = 4 \ln \left(\frac{4x}{5000-x} \right)$

$$\rightarrow 7.5 = \ln \left(\frac{4x}{5000-x} \right)$$

$$e^{7.5} = \frac{4x}{5000-x}$$

$$(5000-x)e^{7.5} = 4x$$

$$5000e^{7.5} - xe^{7.5} = 4x$$

$$5000e^{7.5} = 4x + xe^{7.5}$$

$$5000e^{7.5} = x(4 + e^{7.5})$$

$$x = \frac{5000e^{7.5}}{(4 + e^{7.5})} = 4988.96 \dots$$

$$= 4989 \text{ rabbits infected}$$